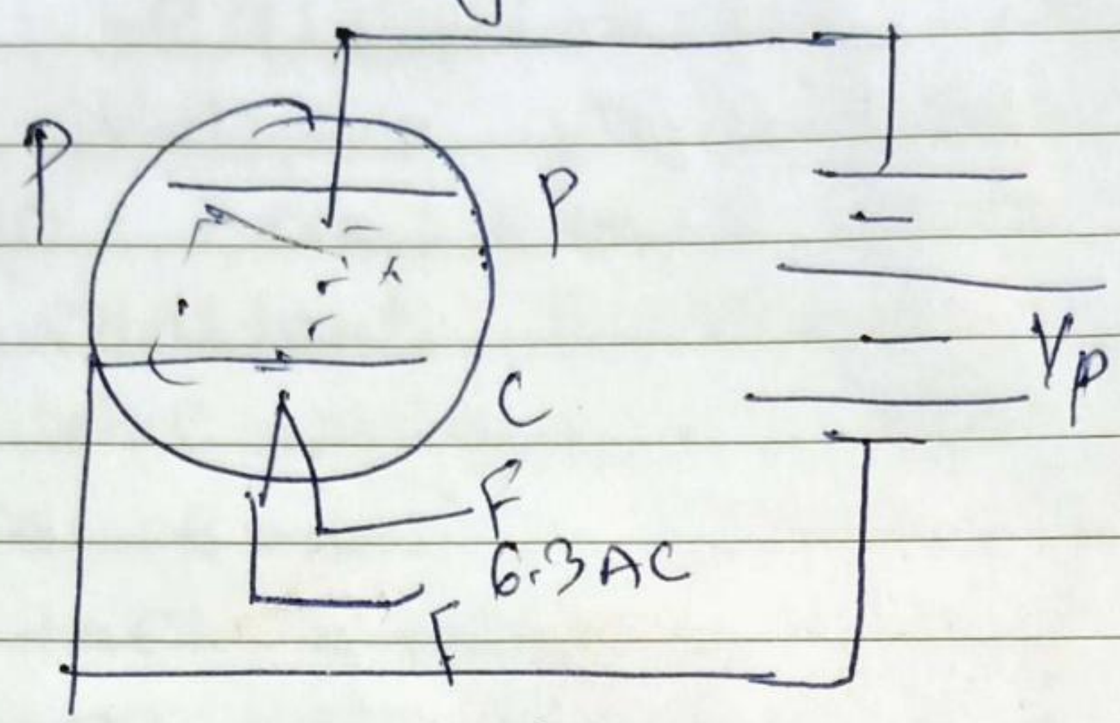
Child langmuir law →



Diode

when diode (vacuum) is operated in space charge limited region, the plate cur I_p is directly proportional to the three ha power of the plate voltage V_p. ie.

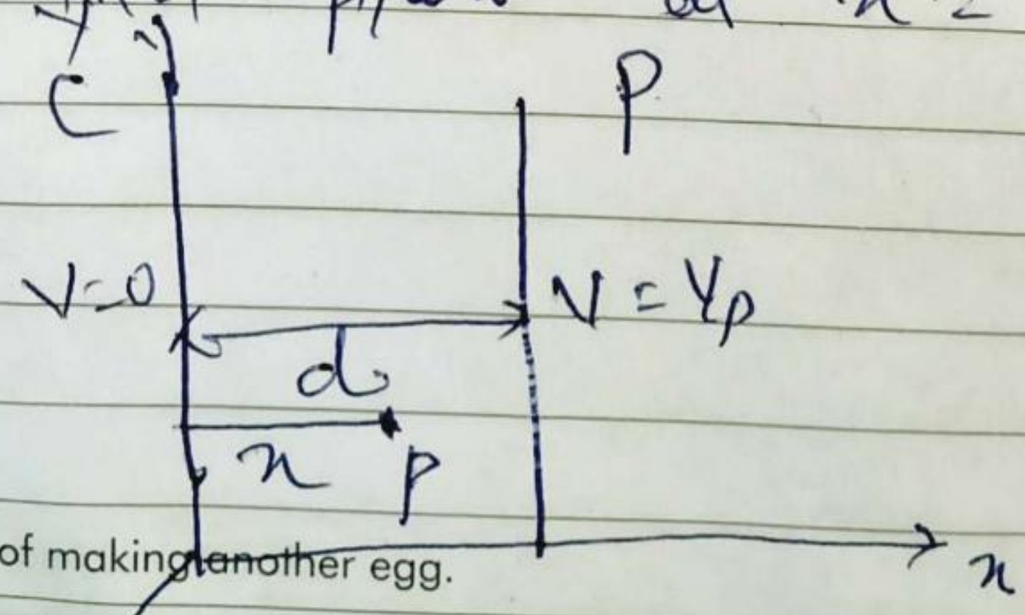
$$I_p \propto V_p^{3/2}$$

$$I_p = k V_p^{3/2}$$

$$\log I_p = \log k + 3/2 \log V_p$$

This is known as child langmuir's l

Derivation → ~~tet~~ Diode contain two p cathode and anode. V_p plate-Voltage space charge limited region. cathode is a $x=0$ and plate at $x=d$.



When electric supply through filament f-3 A.C the cathode heated and electrons emitted from cathode surface and it initial velocity at cathode $v = 0$.

let V = Pot at point P at distance x from the cathode.

An electron of (mass m, charge e) moving from the cathode to the point P will gain energy eV. So K.E of electron $= \frac{1}{2} mu^2$

$$eV = \frac{1}{2} mu^2$$

$$2eV = mu^2 \qquad \boxed{u^2 = \frac{2eV}{m}}$$

which is velocity of electron at P.

n = no of electrons per unit volume at P

ne = ρ = charge density at P.

The current density $J = nev = \rho v$

From law of electrodynamics

$$\frac{d^2V}{dx^2} = \rho / \epsilon_0$$

where ϵ_0 = permittivity of free-space

$$\frac{d^2V}{dx^2} = \frac{J}{\epsilon_0}\sqrt{\frac{m}{2eV}}$$

$$\frac{d^2V}{dx^2} = \frac{J}{\epsilon_0}\left(\frac{m}{2e}\right)^{1/2} V^{-1/2}$$

multiplying both sides by $2\frac{dV}{dx} \cdot dx$

$$2\frac{dV}{dx}dx \frac{d^2V}{dx^2} = \frac{J}{\epsilon_0}\left(\frac{m}{2e}\right)^{1/2} V^{-1/2} \cdot 2 \cdot \frac{dV}{dx} dx$$

integrating it

$$\int 2\frac{dV}{dx}\frac{d^2V}{dx^2} dx = \int \frac{2J}{\epsilon_0}\left(\frac{m}{2e}\right)^{1/2} V^{-1/2} dV$$

$$\left(\frac{dV}{dx}\right)^2 = \frac{4J}{\epsilon_0}\left(\frac{m}{2e}\right)^{1/2} V^{1/2} + c \text{ integration const}$$

when $V = 0$ $dV/dx = 0$ $c = 0$

$$\frac{dV}{dx} = 2\left(\frac{J}{\epsilon_0}\right)^{1/2}\left(\frac{m}{2e}\right)^{1/4} V^{1/4}$$

$$V^{-1/4}\, dV = 2\left(\frac{J}{\epsilon_0}\right)^{1/2}\left(\frac{m}{2e}\right)^{1/4} dx$$

Again integrating

$$\int V^{-1/4}\, dV = \int 2\left(J/\epsilon_0\right)^{1/2}\left(\frac{m}{2e}\right)^{1/4} dx$$

$$4/3\, V^{3/4} = 2\left(J/\epsilon_0\right)^{1/2}\left(\frac{m}{2e}\right)^{1/4} x + c'' \rightarrow \text{integration const}$$

$V = 0$ $x = 0$ $c'' = 0$

$$\therefore 4/3\, V^{3/4} = 2\left(J/\epsilon_0\right)^{1/2}\left(\frac{m}{2e}\right)^{1/4} x$$

Squaing in both side

$$\frac{16}{9} V^{3/2} = 4\left(J/\epsilon_0\right)\left(\frac{m}{2e}\right)^{1/2} x^2$$

$V = V_p$ $x = d$.

$$V_p^{3/2} = \frac{9}{4}\left(J/\epsilon_0\right)\left(\frac{m}{2e}\right)^{1/2} d^2$$

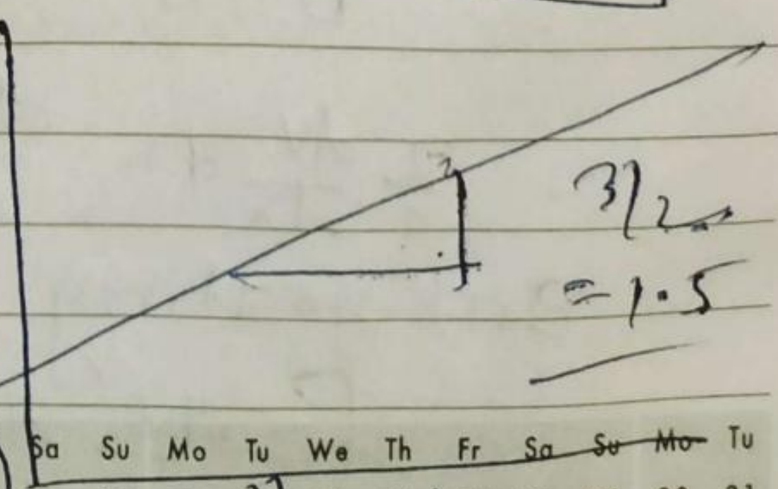
$$J = \frac{4\epsilon_0}{9}\left(\frac{2e}{m}\right)^{1/2} \frac{(V_p)^{3/2}}{d^2}$$

$$J = K V_p^{3/2}$$

$$\boxed{K = \frac{4\epsilon_0}{9d^2}\left(\frac{2e}{m}\right)^{1/2} = \text{const}}$$

$$J_p \propto V_p^{3/2}$$

$$J_p = \boxed{\frac{I_p}{a} \propto V_p^{3/2}}$$

unit area area of cross section of electron of J_p beam unit then

$J_p = I_p$